# Static Analysis of OpenStream Programs

## Using polyhedral techniques to analyze interesting language subsets

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# Parallel languages, runtime execution, and static analysis

Solution(s) for high-level parallel programming?

- Static or dynamic?
- Language constructs or libraries?
- Expressiveness: deterministic (no data races) or deadlock-free?
- How to represent communications and memories? Concurrency?
- Can static optimization help runtime optimizations?
  Buffer sizes, granularity, mapping, ...

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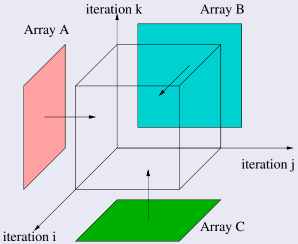
Many approaches:

- "Lower"-level: MPI, OpenCL, Lime, . . .
- Runtime-based: Kaapi, StarPU (with task dep. as in OpenMP 4.0).
- (A)PGAS languages: Co-Array Fortran, UPC, Chapel, X10, . . .
- "Dataflow" languages: KPN, SDF, CSDF, SigmaC, OpenStream, . . .

# Multi-dimensional affine representation of loops and arrays

## Matrix Multiply

```
int i,j,k;
for(i = 0; i < n; i++) {
   for(j = 0; j < n; j++) {
S:     C[i][j] = 0;
       for(k = 0; k < n; k++) {
T:         C[i][j] += A[i][k] * B[k][j];
       }
   }
}
```




## Polyhedral Description — Omega/ISCC syntax

```
Domain := [n]->{S[i][j]: 0<=i,j<n; T[i][j][k]: 0<=i,j,k<n};

Read  := [n]->{T[i][j][k]->A[i][k]; T[i][j][k]->B[k][j];
               T[i][j][k]->C[i][j]};

Write  := [n]->{S[i][j]->C[i][j]; T[i][j][k]->C[i][j]};

Order  := [n]->{S[i][j]->[i][j][0]; T[i][j][k]->[i][j][1][k]};
```

# Limits of automation: often related to the polyhedral model

Polyhedral model: ambiguity with the word “model”.

- Programming/specification model: ☛ Feautrier’s model, Alpha, CRP
- Set of provable techniques under some hypotheses: ☛ SCoP.
- Simplified specification for extracting more general facts.
  ☛ Principle: study a polyhedral subset of a specification/language.

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Examples:

- Uniform loops (simple case to discuss NP-completeness).
- Polyhedral X10 (Yuki, Feautrier, Rajopadhye, Saraswat, PPoPP'13).
- Polyhedral OpenStream (this talk).

# Analyzing X10 through a polyhedral fragment

X10 language developed at IBM, variant at Rice (V. Sarkar)

- PGAS (partitioned global address space) memory principle.
- Parallelism of threads: in particular keywords finish, async, clock.
- No deadlocks by construction but non-determinism.

Polyhedral X10 Yuki, Feautrier, Rajopadhye, Saraswat (PPoPP 2013)

Can we analyze the code for data races?

```
finish {
  for(i in 0..n-1) {
    S1;
    async {
      S2;
    }
  }
}
```

```
clocked finish {
  clocked async {
    S1;
    advance();
  }
  S2;
  advance();
}
```

# Polyhedral X10 dependence analysis

Let $W : A[f(\vec{x})] = \ldots$ be a write and $R : \ldots = A[g(\vec{y})]$ a read in array $A$.

Imperative sequential code (total order)

- $\prec$: disjunction of affine conjunctions (lexicographic order).
- Find the maximal $\vec{x}$, w.r.t. $\prec$, such that:
  - $\vec{x}$ and $\vec{y}$ are legal iterations $\vec{x} \in D_W$; $\vec{y} \in D_R$.
  - $W$ and $R$ access the same element of $A$: $f(\vec{x}) = g(\vec{y})$.
  - $\vec{x} \prec \vec{y}$.

☛ Maximization, gives exact dependence analysis (Feautrier IJPP 1991).

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Polyhedral X10 fragment (add just async: partial order)

- $\prec$ can still be expressed as an incomplete lexicographic order.
- $\vec{x} \prec \vec{y}$ becomes $\neg(\vec{y} \prec \vec{x})$ and $\vec{x} \neq \vec{y}$.
- Maximization now builds a set of (non-comparable) extrema.
- W/W and R/W races can be checked similarly.

☛ Yuki, Feautrier, Rajopadhye, Saraswat (PPoPP 2013).

# What about X10 clocks?

Assume a single clock

$\phi(\vec{x})$ “phase” number of $\vec{x}$: number of advances before $\vec{x}$.

➢ Depends only on $\prec$: can be expressed by polyhedral techniques.

Also, $\vec{x} \prec \vec{y} \Rightarrow \phi(\vec{x}) \leq \phi(\vec{y})$.

Clocks adds ordering constraints

➢ New partial order $\prec_c$: $\vec{x} \prec_c \vec{y}$ iff $\vec{x} \prec \vec{y}$ or $\phi(\vec{x}) < \phi(\vec{y})$.

Detection of races?

➢ Now becomes undecidable (Hilbert 10th problem), Feautrier (2013).

X10: expressiveness + deadlock-free $\Rightarrow$ non-determinism + undecidability

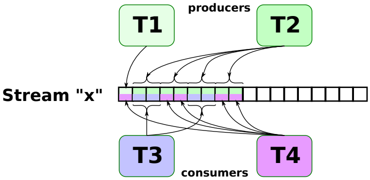
OpenStream: expressiveness + determinism $\Rightarrow$ deadlocks + undecidability.

# Analyzing OpenStream through a polyhedral fragment

```
#pragma omp task output (x) // Task T1
x = ...;
for (i = 0; i < N; ++i) {
  int window_a[2], window_b[3];

  #pragma omp task output (x « window_a[2]) // Task T2
  window_a[0] = ...; window_a[1] = ...;
  if (i % 2) {
    #pragma omp task input (x » window_b[2]) // Task T3
    use (window_b[0], window_b[1]);
  }
  #pragma omp task input (x) // Task T4
  use (x);
}
```

(Pop, Cohen, 2011)

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- Reservation for reads/writes in streams with burst and horizon.
- Single assignment in streams (by construction) + dataflow semantics.

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  if (i % 2) {
    #pragma omp task input (x >> window_b[2]) // Task T3
    use (window_b[0], window_b[1]);
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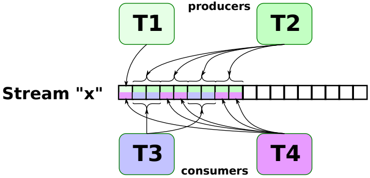


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- Instance of Pop-Cohen CDDF (Control-Driven Data Flow).
- Optimization in Erbium runtime system explored by Pop & Miranda.
- Unlike KPN, streams with multiple inputs/outputs (but deterministic).

# Some properties of polyhedral OpenStream

- The order of activations is the sequential order of the control program.
- Access functions to streams are statically-expressible polynomials.
- Dependence analysis is feasible with tools manipulating polynomials.

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Deadlock characterization iff there is, in the unrolled dependence graph,

- a statement $T$ and an infinite sequence $(i_j)_{j \in N}$ such that $T(i_j)$ depends on $T(i_{j+1})$ and $i_j$ is before $i_{j+1}$ in the activation program;
- a cycle if the program is bounded or follows Kahnian semantics.

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Deadlock detection If access indices can express polynomials of large degrees, deciding the presence of deadlocks is undecidable, and one can build such an OpenStream program.

☛ Consequence: Not clear how to optimize, unless with polynomial computations (cf Feautrier, with Schweighofer/Handelman theorem).

# First ingredient (Feautrier): build multivariate polynomials

$Q(x_1, \ldots, x_n)$: multivariate polynomial, nonnegative integer coefficients.

Write:

- $Q(x) = Q(x_1, x_r)$, $x_1$ first variable.
- $Q^1(x_1, x_r) = Q(x_1 + 1, x_r) - Q(x_1, x_r)$ (first difference)
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  - ☛ Can compute $Q(x)$ with:

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phi = Q(0,x_r);
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- Continue with other variables:

```
phi = Q(0,x_r); // Put new loops
for (i = 0; i < x; i++) {
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# Second ingredient: build the OpenStream structure

```
s, t streams;
for (x in D) {
/* D is the n-dimensional cube
   of size N in the first orthant */
  R1: read Q(x) times in t;
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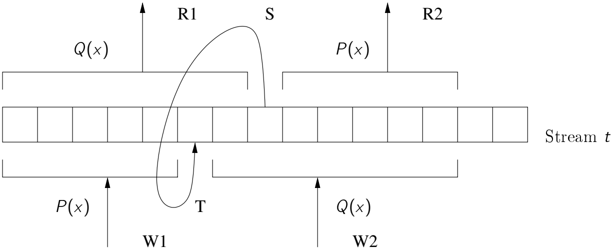


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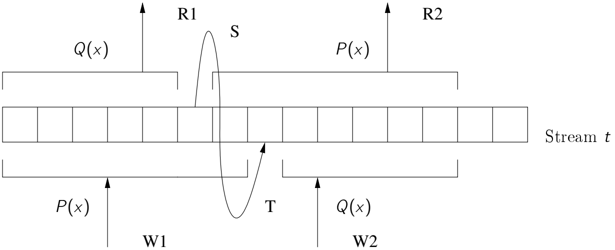


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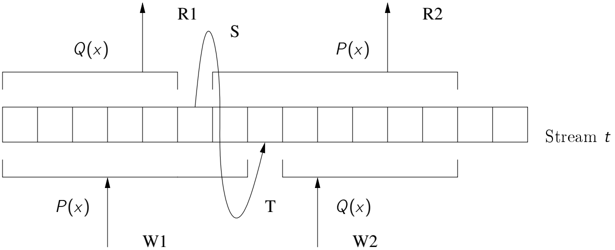


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☛ 10th Hilbert's problem:

- $R(x) = 0$ iff $R^+(x) = R^-(x)$.
- $R(x) = 0$ iff $R^2(x) \leq 0$.

# Take-home messages

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## About parallel languages

- What do you prefer: deadlocks or races?
- How to express link between user/compiler and compiler/runtime?
- Parallel constructs can help dependence analysis (cf V. Sarkar).

☛ Towards the analysis of parallel languages, with better user/compiler and compiler/runtime interactions.